Tutorial Worksheet 11

1) (a) Prove directly, that the following relation, on the set of integers, is an equivalence relation.

$a \equiv b$ if and only if $a - b$ is divisible by 4.

(b) Show that if two integers satisfy the relation in part (a), then they have the same remainder when divided by 4 (refer to Theorem 6.1.2 and Exercise 6.4.7).

2) Define, on the set of integers, the following equivalence relation.

$k \sim l$ if and only if $|k| = |l|$.

(a) Prove that the above relation is indeed an equivalence relation.

(b) Describe the equivalence classes for this relation.

3) Let $f : A \to B$ be an arbitrary function. Prove that the relation

$x \sim y$ if and only if $f(x) = f(y)$,

on the set A, is an equivalence relation.